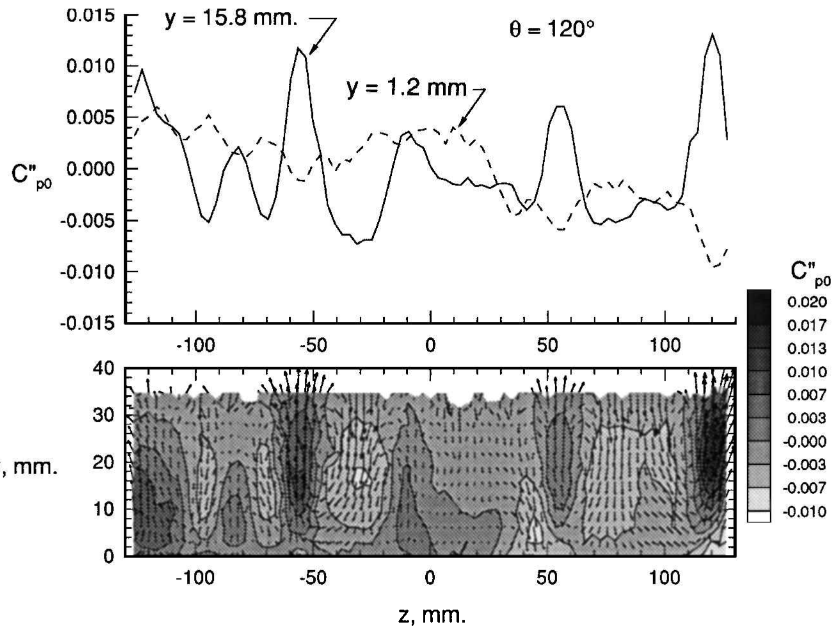


Fig. 4 Measured crossflow velocity vectors and total pressure in the $\theta = 120$ deg plane on the circular cylinder [conditions as in Fig. 3; $C_{P0} = (P_0 - p_\infty)/0.5\rho U_J^2$; $''$ denotes the difference between local and spanwise ensemble-average values].

vortices whose size and strength were nevertheless comparable to those with the original dirty slot lip.

The evidence indicates that streamwise vortices develop as a result of rapid amplification of small initial distortions introduced into the thin, centrifugally unstable, free shear layer that forms the upper boundary of the flow leaving the slot. Furthermore, the sensitivity to distortions appears to be such that streamwise vortices would most probably be present in any practical realization of tangential blowing over surfaces with substantial convex curvature. Values of $U_J b/\nu$ in the present and earlier[3,4] experiments are as much as an order of magnitude lower than in practical applications. However, the experimental flows were fully turbulent, with longitudinal turbulence intensities of about 12% at $y = y_{1/2}$ (Ref. 3). The observed flow behavior is thus expected to be representative of that of turbulent convex wall jets regardless of the value of the Reynolds number.

Acknowledgments

This work was supported by a research grant awarded by the Natural Sciences and Engineering Research Council of Canada. The first author also gratefully acknowledges a scholarship awarded by the Canadian International Development Agency during the course of the work.

References

[1]Kind, R. J., and Maull, D. J., "An Experimental Investigation of a Low-Speed Circulation-Controlled Aerofoil," *Aeronautical Quarterly*, Vol. 19, Pt. 2, 1968, pp. 170–182.

[2]White, F. M., *Viscous Fluid Flow*, 2nd ed., McGraw–Hill, New York, 1991, pp. 366–368.

[3]Kind, R. J., "A Proposed Method of Circulation Control," Ph.D. Dissertation, Univ. of Cambridge, Cambridge, England, UK, June 1967.

[4]Kind, R. J., Gooden, K., and Dvorak, F. A., "Measurements of Flows with Tangential Injection and Comparison with Prediction Methods," *AIAA Journal*, Vol. 17, No. 7, 1979, pp. 730–735.

[5]Guitton, D. E., "Some Contributions to the Study of Equilibrium and Non-Equilibrium Wall Jets over Curved Surfaces," Ph.D. Thesis, McGill Univ., Montreal, PQ, Canada, 1970.

[6]DeCecco, S., "The Aerodynamics of Turbine Blades with Tip Damage," M.E. Thesis, Dept. of Mechanical and Aerospace Engineering, Carleton Univ., Ottawa, ON, Canada, April 1995.

[7]Fekete, G. I., "Coanda Flow in a Two-Dimensional Wall Jet on the Outside of a Circular Cylinder," Mechanical Engineering Dept., Rept. 63-11, McGill Univ., Montreal, PQ, Canada, 1963.

R. M. C. So
Associate Editor

Two-Layer Approach Combining Reynolds Stress and Low-Reynolds-Number k–ε Models

W. D. Hsieh* and K. C. Chang†
National Cheng-Kung University,
Tainan 701, Taiwan, Republic of China

I. Introduction

THE modeling of low-Reynolds-number (LRN) turbulence in the immediate vicinity of a wall has become one of the most important issues in turbulence modeling work. A number of LRN versions of the Reynolds stress models (RSM) have been developed recently.[1–3] Each new LRN RSM is more complex mathematically than the original RSM version. Furthermore, these LRN RSM have mostly been tested with simple isothermal flows. The generality of these models to the simulation of complex flows with/without wall heat/mass transfer is questionable. An alternative approach that was widely adopted in the past decade is the two-layer model, which depicts the overall turbulent flowfield by dividing it into the near-wall and fully turbulent flow zones. The basic idea of this alternative approach is that even a complex (more anisotropic) flow would become a simple (less anisotropic) type while being located within the near-wall region. This kind of turbulent model includes the k–ε/one-equation model (OEM),[4–7] algebraic stress models (ASM)/OEM,[8] RSM/OEM,[1] and ASM/LRN k–ε model.[8,9] Although two-layer models have been applied to many kinds of turbulent flows including nonisothermal cases, their performances were not always plausible. The reasons are as follows: The two-layer models are a combination of two turbulence models applicable individually to the high- and low-Reynolds-number flow regimes. The shortcomings of the k–ε models and the ASM in applications on complex flows are well known.[2] The incapability of the one-equation turbulence model to

Received Feb. 23, 1998; revision received Oct. 22, 1998; accepted for publication Oct. 22, 1998.

*Research Associate, Institute of Aeronautics and Astronautics.
†Professor, Institute of Aeronautics and Astronautics. E-mail: kcc@mail.iaa.ncku.edu.tw. Senior Member AIAA.

simulate the flows in which the flow separation phenomena occur is also well known.

A two-layer model that is constructed by an RSM and an LRN k–ε model is proposed in this Note to overcome the deficiencies of the existing two-layer models. Demonstration of the proposed two-layer model performance is implemented through two (one isothermal and the other nonisothermal) recirculating flows in a pipe-expansion configuration.

II. Turbulence Models

Two unknown correlations, i.e., the Reynolds stress $-\rho\overline{u_i'u_j'}$ and the turbulent heat flux $-\rho\overline{u_i'T'}$, appear in the Reynolds-averaged equations of momentum and energy and need modeling.

LRN k–ε Model

The LRN k–ε model developed in our previous work[10] [the Chang–Hsieh–Chen (CHC) model] is used for demonstration. The Reynolds stress is expressed using the Boussinesq eddy-viscosity approximation with a scalar eddy viscosity defined as

$$\mu_t = \frac{\rho C_\mu f_\mu k^2}{\varepsilon} \tag{1}$$

where f_μ is the damping function accounting for the viscosity-affected factor. The turbulent heat flux is modeled using the eddy viscosity hypothesis (EVH), which appeals to the classical analogy between momentum and energy transport, given as

$$-\rho\overline{u_j'T'} = \frac{\mu_t}{\sigma_t}\frac{\partial T}{\partial x_j} \tag{2}$$

The CHC model has been shown to be able to yield correct near-wall limiting behaviors and to avoid the singular problem in the neighborhood of reattachment points as applied to the flows in the investigated configuration.[10,11]

RSM

The Reynolds stress transport equations can be symbolically written as follows:

$$\frac{\partial}{\partial x_k}(\rho u_k\overline{u_i'u_j'}) = D_{ij}^V + D_{ij}^T + P_{ij} + \Phi_{ij} - \rho\varepsilon_{ij} \tag{3}$$

Here the production term P_{ij} and the molecular diffusion term D_{ij}^V are exact. The turbulent diffusion term D_{ij}^T is modeled using the generalized gradient diffusion hypothesis[12] as

$$D_{ij}^T = \frac{\partial}{\partial x_k}\left[\left(\rho C_s f_\mu \frac{k}{\varepsilon}\overline{u_k'u_l'}\right)\frac{\partial\overline{u_i'u_j'}}{\partial x_l}\right] \tag{4}$$

Note that the damping function f_μ is included in the formulation of D_{ij}^T because the matching interface between the two flow zones, to which the RSM and the LRN k–ε model individually apply, is chosen at the position of $f_\mu = 0.9$. In other words, the zone in which the RSM applies still covers partially the low-Reynolds-number flow regime. Modeling of the pressure-strain correlation term Φ_{ij} follows that used previously by Nikjooy and Mongia.[13] Because the RSM model is applied only to the fully turbulent flow zone in the two-layer approach, wall correction to the pressure-strain correlation term is not considered. Local isotropy is made to the viscous dissipation term because most of the viscous dissipation occurs at small scales in the flows with high Reynolds numbers.

The turbulent heat flux is modeled similarly to the one proposed by Daly and Harlow[12]:

$$-\rho\overline{u_j'\theta} = \rho C_T f_\mu \frac{k}{\varepsilon}\overline{u_j'u_k'}\frac{\partial T}{\partial x_k} \tag{5}$$

The preceding formulation was able to yield a more accurate representation of heat fluxes in certain anisotropic flowfields in which the modeling of the turbulent heat fluxes based on the EVH [Eq. (2)] performed poorly.[5,8,9] Furthermore, the formulation of Eq. (5) reduces the complexity of the equation system at the present stage for demonstration of the proposed two-layer model by avoiding transport equations for the additional turbule quantities.

III. Numerical Analysis

The finite volume method incorporated with the SIMPLER algorithm and the orthodox QUICK scheme[14] is used to solve numerically the transport equations. A grid mesh consisting of 110×90 nonuniformly distributed grid nodes is used for the calculations. Because of concern for stability in the numerical calculation, it is necessary to introduce pseudodiffusion terms (represented by the apparent viscosities multiplied by their corresponding velocity gradients) into the Reynolds-averaged momentum equations using the RSM. This approach was found to bring about a significant increase in the convergent rate. Formulation of the apparent viscosity follows the approach proposed by Hogg.[15]

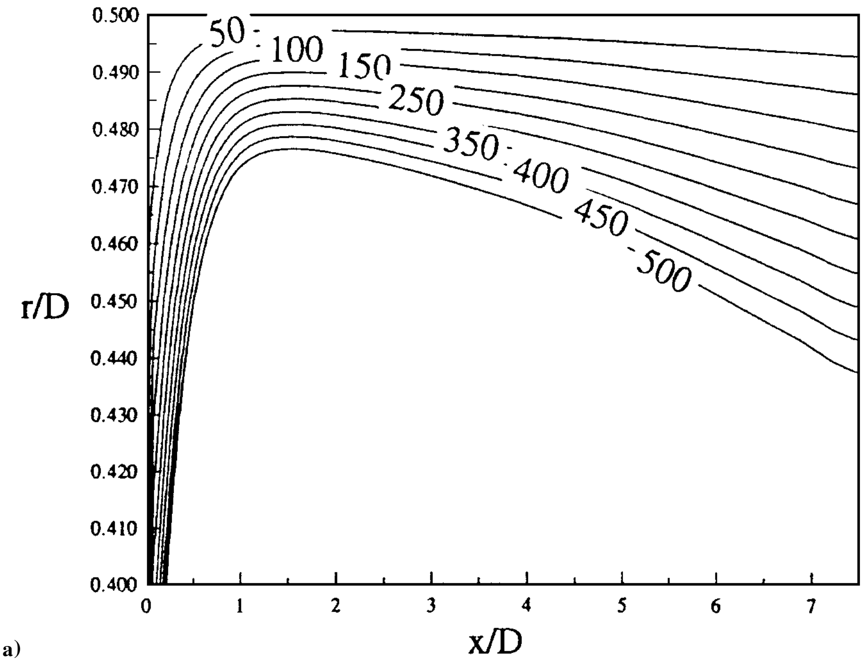
Determination of the matching interface between the two flow zones to which the LRN k–ε model and the RSM are separately applied is an important issue in the two-layer modeling work. R_k ($=\sqrt{k}y/\nu$) was widely adopted as the parameter to determine the matching interface in the existing two-layer models.[4,6,7] For example, the values of R_k used for designating the matching interface are between 200 and 250 in the cases of isothermal flows,[4–7] whereas the values of R_k range from 50 to 100 in the cases of nonisothermal flows.[8,9] Obviously, these R_k values are case dependent. Instead of R_k, the model function f_μ, which is expressed in terms of R_k and an additional parameter R_t ($=k^2/\nu\varepsilon$; Ref. 10), is now chosen as the parameter to determine the matching interface. As for the original design of the f_μ function, $f_\mu = 1$ in the fully turbulent zone, whereas $f_\mu < 1$ in the viscosity-affected layer. A criterion of $f_\mu = 0.9$ is used to determine the matching interface and will be justified later.

IV. Results and Discussion

Demonstration using the proposed two-layer model is first made for an isothermal, axisymmetrical flow in a pipe-expansion configuration with the expansion ratio of 1.904 and the Reynolds number (based on the downstream diameter) of 1.6×10^5, which was experimentally investigated by Durret et al.[16] No wall boundary conditions are needed for u'^2, v'^2, w'^2, and $\overline{u'v'}$ because the LRN k–ε is used for the calculation in the near-wall region. Other boundary conditions are specified in the usual way (for details see Ref. 17). As anticipated, the performance of the two-layer (hybrid RSM/CHC) model is better than that of the CHC (k–ε turbulence) model in the fully turbulent zone. For reasons of brevity, this part of the results is not presented here and can be found in Ref. 17. Here special interest is addressed to the model performance in the near-wall region. The reattachment lengths predicted through the two-layer and CHC models are 1.94 and 1.92D, respectively, as compared to the experimental data of 1.97D (Ref. 16). There were no other near-wall data available for model verification reported by Durret et al.[16] Figure 1 displays two contour maps of the R_k parameter and the f_μ function, obtained with the two-layer model, in the near-wall region. A comparison reveals that the contour of $f_\mu = 0.9$, which was set as the matching interface in the work, is closer to the wall (located in the more viscosity-affected region) than the contours of $R_k = 200$–250, which were usually adopted as the matching interfaces by the existing two-layer models.

To examine further the near-wall performance of the hybrid model, a nonisothermal case that was experimentally investigated by Zemanick and Dougall[18] with a similar configuration but with the expansion ratio of 1.86 and $Re = 9620$ is tested as well. Uniform distributions of the mean inlet temperature of 300 K and of the wall heat flux, which were prescribed in the experiment,[18] are specified as part of the boundary conditions. Two Nu distributions along the wall, predicted with the two-layer and CHC models and normalized by a quantity of Nu_{fd} ($=0.023Re^{0.8}\,Pr^{0.4}$) representing the fully developed Nu value of a straight-pipe flow at the same fluid properties, are presented in Fig. 2. The comparison reveals that the two Nu predictions using the CHC and two-layer models do not differ much from each other and both are in fair agreement with the measurements.

Figure 3 displays two contour maps of the R_k parameter and the f_μ function, obtained with the two-layer model, in the near-wall region. A comparison shows that the contour of $f_\mu = 0.9$, designated for the matching interface in the work, falls into the zone bounded



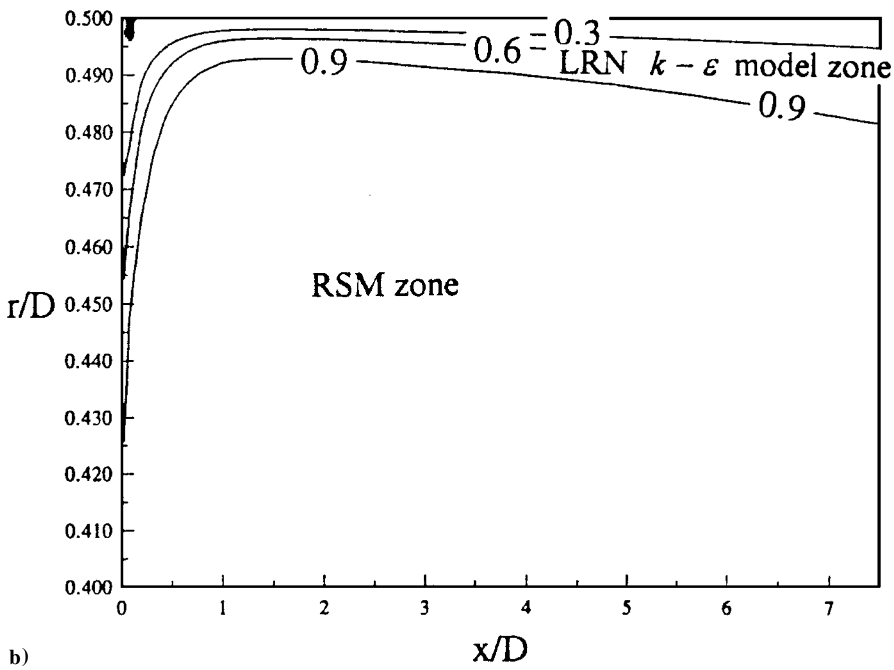


Fig. 1 Contour diagrams of the a) R_k and b) f_μ parameters in the near-wall region of the isothermal case.

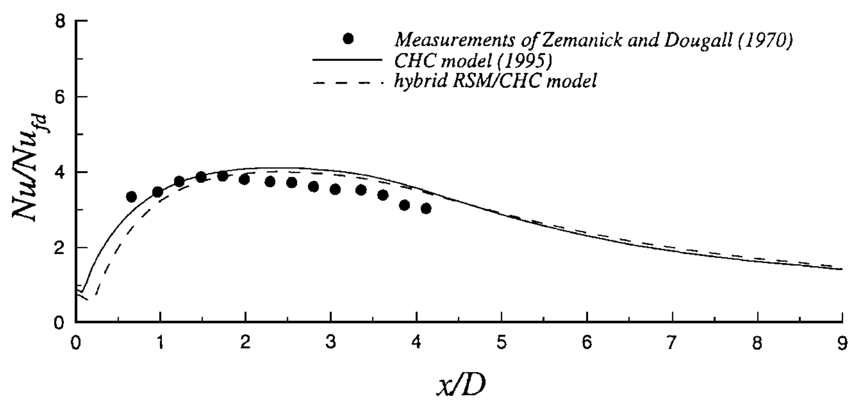


Fig. 2 Comparison of the Nusselt number distributions along the wall obtained through the CHC and two-layer models with the measurements in the nonisothermal case.

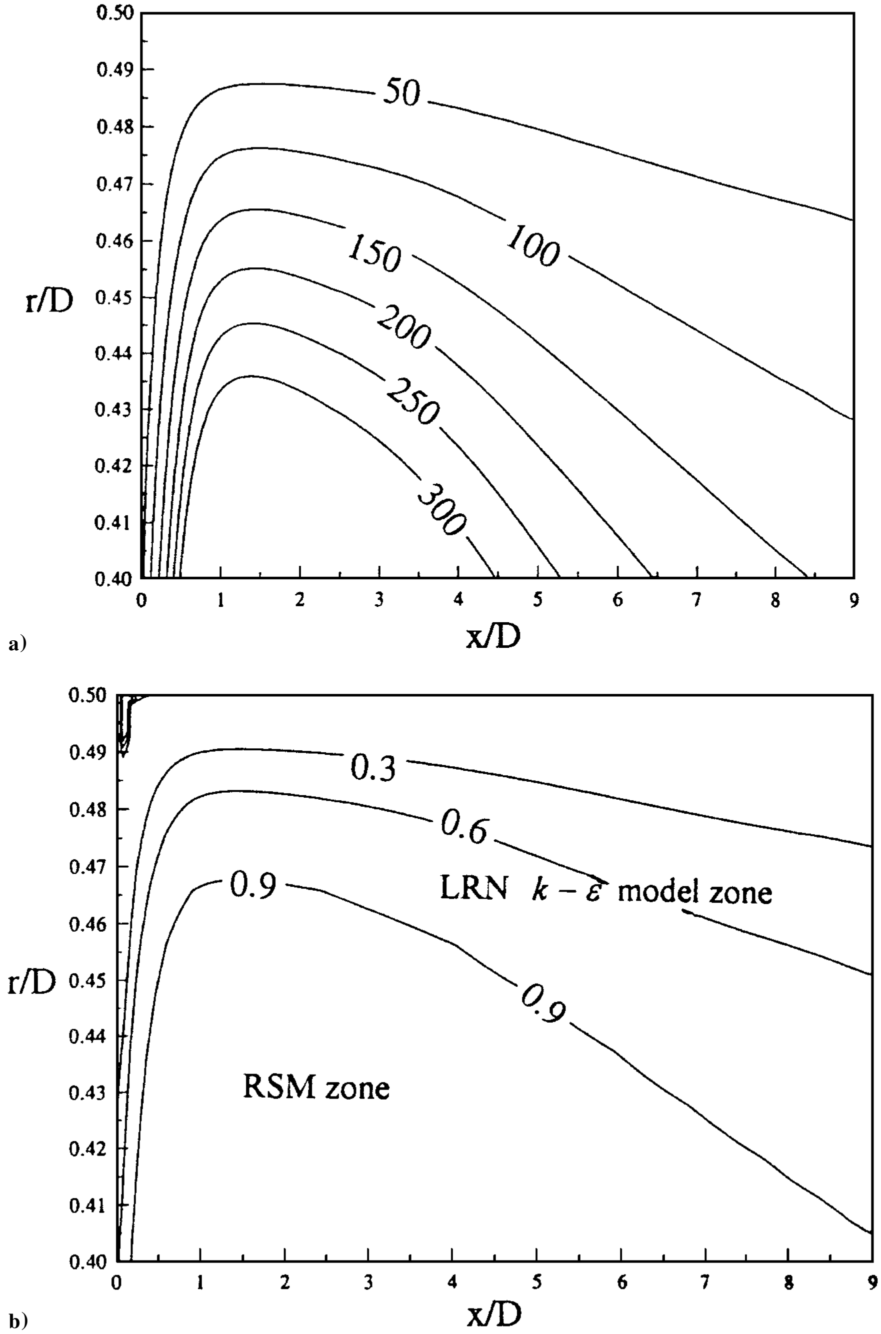


Fig. 3 Contour diagrams of the a) R_k and b) f_μ parameters in the near-wall region of the nonisothermal case.

by R_k contours of 100 and 150. In contrast, the usual values of R_k used for designating the matching interface in the nonisothermal studies using the two-layer models[8,9] ranged from 50 to 100. As indicated in Fig. 3, the positions with $R_k = 50$–100 are located deeper inside the viscosity-affected layer in which the applicability of the conventional (high-Reynolds-number) RSM might be questionable.

V. Conclusions

A two-layer turbulence model combining an RSM and an LRN k–ε model is proposed. Performance demonstration using the proposed two-layer model is implemented through an isothermal and a nonisothermal flow in pipe-expansion configuration. It is evident that the proposed two-layer model can retain, at least, the same performance level in the near-wall region while yielding better performance in the fully turbulent zone[17] as compared to what was made with the CHC model, which is a pure LRN k–ε model. Determination of the matching interface between the two layers is suitable by using the criterion of $f_\mu = 0.9$. Another advantage of using this two-layer model is that either part of the RSM or the LRN k–ε model adopted here is readily replaced once a better version of the RSM or the LRN k–ε model is available.

Acknowledgment

This work was supported by National Science Council of the Republic of China under Contract NSC 86-2212-E006-100.

References

[1]Davidson, L., "Prediction of the Flow Around an Airfoil Using a Reynolds Stress Transport Model," *Journal of Fluids Engineering*, Vol. 117, No. 1, 1995, pp. 50–57.

[2]Speziale, C. G., and Gatski, T. B., "Assessment of Second-Order Closure Models in Turbulent Shear Flows," *AIAA Journal*, Vol. 32, No. 10, 1994, pp. 2113–2115.

[3]So, R. M. C., Lai, Y. G., and Zhang, H. S., "Second-Order Near-Wall Turbulence Closures: A Review," *AIAA Journal*, Vol. 29, No. 11, 1991, pp. 1819–1835.

[4]Patel, V. C., and Yoon, J. Y., "Application of Turbulence Models to

Separated Flow over Rough Surfaces," *Journal of Fluids Engineering*, Vol. 117, No. 2, 1995, pp. 234–241.

[5]Jansson, L. S., Davidson, L., and Olsson, E., "Calculation of Steady and Unsteady Flows in a Film-Cooling Arrangement Using a Two-Layer Algebraic Stress Model," *Numerical Heat Transfer*, Pt. A, Vol. 25, No. 3, 1994, pp. 237–258.

[6]Mohammadi, B., "Complex Turbulent Compressible Flow Computation Using Two-Layer Approach," *International Journal for Numerical Methods in Fluids*, Vol. 15, No. 7, 1992, pp. 747–771.

[7]Chen, H. C., and Patel, V. C., "Near-Wall Turbulence Models for Complex Flows Including Separation," *AIAA Journal*, Vol. 26, No. 6, 1988, pp. 641–648.

[8]Yap, C., "Turbulent Heat and Momentum Transfer in Recirculating and Impinging Flows," Ph.D. Thesis, Dept. of Mechanical Engineering, Univ. of Manchester, Manchester, England, UK, 1987.

[9]Davidson, L., "Second-Order Corrections of the k–ε Model to Account for Non-Isotropic Effects Due to Buoyancy," *International Journal of Heat and Mass Transfer*, Vol. 33, No. 12, 1990, pp. 2599–2608.

[10]Chang, K. C., Hsieh, W. D., and Chen, C. S., "A Modified Low-Reynolds-Number Turbulence Model Applicable to Recirculating Flow in Pipe Expansion," *Journal of Fluids Engineering*, Vol. 117, No. 3, 1995, pp. 417–423.

[11]Hsieh, W. D., and Chang, K. C., "On Calculation of Turbulent Heat Transfer in Pipe-Expansion Flows," *International Journal of Heat and Mass Transfer*, Vol. 39, No. 18, 1996, pp. 3813–3822.

[12]Daly, B. J., and Harlow, F. H., "Transport Equation in Turbulence," *Physics of Fluids*, Vol. 13, No. 11, 1970, pp. 2634–2649.

[13]Nikjooy, M., and Mongia, H. C., "A Second-Order Modeling Study of Confined Swirling Flow," *International Journal of Heat and Fluid Flow*, Vol. 12, No. 1, 1991, pp.12–19.

[14]Hsieh, W. D., and Chang, K. C., "Turbulent Flow Calculation with Orthodox QUICK Scheme," *Numerical Heat Transfer*, Pt. A, Vol. 30, No. 6, 1996, pp. 589–604.

[15]Hogg, S. I., "Second-Moment Closure Calculation of Strong-Swirling Confined Flows with and Without Density Variations," Ph.D Thesis, Dept. of Mechanical Engineering, Univ. of Manchester, Manchester, England, UK, 1988.

[16]Durret, R. P., Stevenson, W. H., and Thompson, H. D., "Radial and Axial Turbulent Flow Measurements with an LDV in an Axisymmetrical Sudden Expansion Air Flow," *Journal of Fluids Engineering*, Vol. 110, No. 4 , 1988, pp. 367–372.

[17]Hsieh, W. D., "A Hybrid RSM/LRN k–ε Turbulence Model for Complex Flows," Ph.D Thesis, Inst. of Aeronautics and Astronautics, National Cheng-Kung Univ., Taiwan, ROC, Oct. 1996.

[18]Zemanick, P. P., and Dougall, R. S., "Local Heat Transfer Downstream of Abrupt Circular Channel Expansion," *Journal of Heat Transfer*, Vol. 92, No. 2 , 1970, pp. 53–60.

C. G. Speziale
Associate Editor